Exercice 4

Dessin Ω :

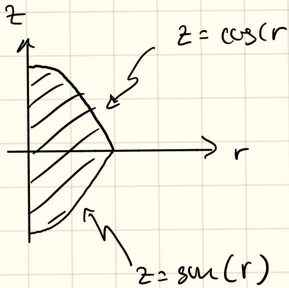
$\Omega = \{ (x,y,z) \in \mathbb{R}^3 : x^2+y^2 < \frac{\pi^2}{4} \ , \cos(\sqrt{x^2+y^2}) \}$

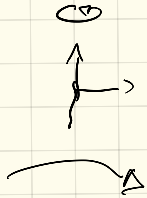
a une symétrie cylindrique:

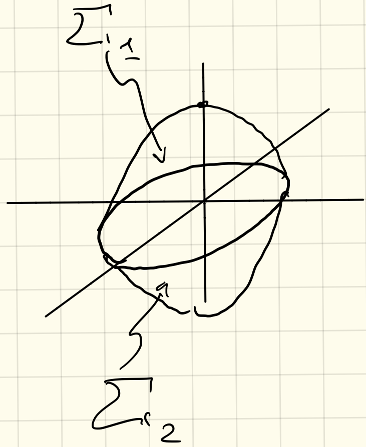
$(x,y,z) = (r\cos\theta, r\sin\theta, z) \quad r \in [0, \frac{\pi}{2}], \theta \in [0, 2\pi]$

$z \in [-\cos(r), \cos(r)]$

(Noter que $r \in [0, \pi/2] \Rightarrow \cos(r) \geq 0$.







Calcul normale :

$$\alpha_r \wedge \alpha_\theta = \begin{vmatrix} e_1 & e_1 & e_3 \\ \cos\theta & \sin\theta & -\sin(r) \\ -r\sin\theta & r\cos\theta & 0 \end{vmatrix} = \begin{pmatrix} r\cos(\theta)\sin(r) \\ r\sin(\theta)\sin(r) \\ r \end{pmatrix}$$

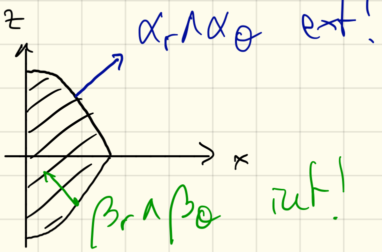
$$\beta_r \wedge \beta_\theta = \begin{vmatrix} e_1 & e_1 & e_3 \\ \cos\theta & \sin\theta & \sin(r) \\ -r\sin\theta & r\cos\theta & 0 \end{vmatrix} = \begin{pmatrix} -r\cos(\theta)\sin(r) \\ -r\sin(\theta)\sin(r) \\ r \end{pmatrix}$$

En testant en $r = \frac{\pi}{4}$, $\theta = 0$,

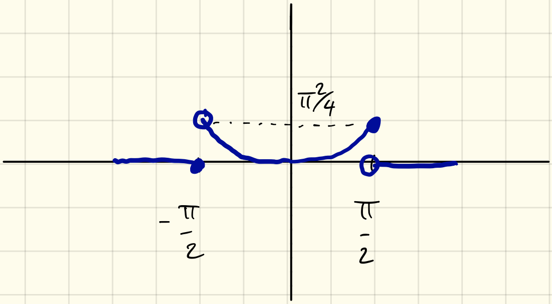
$$\alpha(r,\theta) = \left(\frac{\pi}{4}, 0, \frac{\sqrt{2}}{2}\right), \qquad \alpha_r \wedge \alpha_\theta = \begin{pmatrix} \frac{\pi}{2} \cdot \frac{\sqrt{2}}{2} \\ 0 \\ \frac{\pi}{4} \end{pmatrix}$$

$$\beta(r,\theta) = \left(\frac{\pi}{4}, 0, \frac{\sqrt{2}}{2}\right), \qquad \beta_r \wedge \beta_\theta = \begin{pmatrix} -\frac{\pi}{2} \cdot \frac{\sqrt{2}}{2} \\ 0 \\ \frac{\pi}{4} \end{pmatrix}$$

$\boxed{y = 0}$



Exercice 5



f est continue $\forall x \neq \pm\frac{\pi}{2}$, donc $Ff(x) = f(x)$.

pour $x = \pm\frac{\pi}{2}$, $Ff(x) = \dfrac{f(x-0) + f(x+0)}{2} = \dfrac{\pi^2/4 + 0}{2} = \dfrac{\pi^2}{8}$

$$\Rightarrow \quad Ff(x) = \begin{cases} 0 & \text{si } -\pi < x < -\pi/2 \\ \pi^2/8 & \text{si } x = -\frac{\pi}{2} \\ x^2 & \text{si } -\pi/2 < x < \frac{\pi}{2} \\ \pi^2/8 & \text{si } x = \frac{\pi}{2} \\ 0 & \text{si } \frac{\pi}{2} < x \leq \pi \end{cases}$$